- 1. Assume you toss your thumb tack 10 times. Let  $X_i$  denote the result of the *i*'th throw. We set  $X_i = 1$  if the pin pointed upwards and  $X_i = 0$  otherwise.
  - (a) Define the random variable S that denotes the number of times a pin pointed upward as a mathematical term combining the  $X_i$ .
  - (b) Use notations with these random variables  $X_i$  to define the following events:
    - A: "The pin pointed downward in the 1st and 4st throw."
    - B: "The pin pointed upward in 5 out of 10 throws."
    - C: "The pin pointed upward in the first 3 throws, and 4 times in total."
    - D: "The pin pointed downward more than once."
  - (c) Assume the probability that the pin points upward after a throw is 0.4. What is the distribution of S? Calculate the probabilities of the events A to D.
- 2. The manufacturer of the thumb tacks claims that the thumb tacks have been designed in a way such that they can also be used as a random generator, with the probability of the pin pointing upwards being 0.5.
  - (a) Perform experiments and calculations to check this. If you do not have a thumb tack, use data from one of the lines in http://evol.bio.lmu.de/\_statgen/ maths/ws2021/thumbtack\_2020.csv (or http://evol.bio.lmu.de/\_statgen/ maths/ws2022/thumbtack\_2022.csv if already available). You can read the data directly into R with the command

read.csv("http://evol.bio.lmu.de/\_statgen/maths/ws2022/thumbtack\_2022.csv").
As part of the test, answer the following questions:

- i. Define your null hypothesis  $H_0$ .
- ii. What is the alternative hypothesis?
- iii. What is the distribution of S under the null hypothesis?
- iv. Which possible results are as extreme as or more extreme than your observation?
- v. Calculate the *p*-value.
- vi. Using a significance level of 5%, what does your test indicate? Formulate the results as a sentence.
- (b) Discuss to what extent you can substantiate or even proof that the claim of the manufacturer is right or wrong.
- (c) Do the same for the case that the manufacturer only claims that the probability is in the range between 0.45 and 0.55.
- 3. Test for the data on http://evol.bio.lmu.de/\_statgen/maths/ws2021/thumbtack\_ 2020.csv(or http://evol.bio.lmu.de/\_statgen/maths/ws2022/thumbtack\_ 2022.csv if already available) perform (separate) statistical tests for the following two null hypotheses:
  - (a) If you average over all thumb tacks the probability to point upwards, the result is 0.5.
  - (b) All thumb tacks have the same probability to point upwards.