Exercise 1: The vector \( \mathbf{X} = \begin{pmatrix} X_1 \\ X_2 \end{pmatrix} \) is normally distributed with \( \mathbb{E}X_1 = 0.5 \) and \( \mathbb{E}X_2 = 1 \). Further, the standard deviations of \( X_1 \) and \( X_2 \) are 0.2 and 0.4, respectively, and their correlation is \(-0.1\). Calculate the expectation vector and the covariance matrix of the vector

\[
\mathbf{Y} = \begin{pmatrix} 2 & 0.5 \\ 1 & -1 \end{pmatrix} \cdot \mathbf{X}
\]

Exercise 2: The average values of two quantitative traits have been determined for four species of known phylogeny, as shown in the figure.

(a) Calculate the correlation of the traits (without taking the phylogeny into account).

(b) Calculate the independent contrasts for each of the two traits. Do this step by step without using special software.

(c) Calculate the correlation of the independent contrasts of the two traits.

(d) Is the correlation between the traits significant? Does the answer to this question depend on whether you take the phylogeny into account?

Exercise 3: The files QuantTraitsA.csv, QuantTraitsB.csv, and QuantTraitsC.csv contain datasets of quantitative traits for species whose phylogeny is given in the file QuantTraits_Tree.txt. Do you find evidence for selection and/or for correlated evolution of some of the traits?

Exercise 4: Write a computer program or an R script that reads a tree in newick format and simulates the evolution of quantitative traits along the branches of the tree. The output should be the values of the traits at the tips of the tree.

Exercise 5: Explore with simulated data how robust posterior probabilities given by BEAST are against misspecification of priors, substitution models and relaxed-clock models.

Exercise 6: The two values 1.0 and 1.1 are independently drawn from the same normal distribution \( \mathcal{N}(\mu, \sigma^2) \) with unknown \( \mu \) and unknown \( \sigma^2 \). We would like to know whether \( \mu = 0 \).

(a) Apply a \( t \)-test to address this question.

(b) Compute Bayes-factors to compare the following two hypotheses
$H_0 : \mu = 0$ and $\sigma^2$ has a uniform prior on $[0, s]$

$H_A : \mu$ has a uniform prior on $[-m, m]$ and $\sigma^2$ has a uniform prior on $[0, s]$

How does the decision for one or the other model depend on $m$ and $s$?