Exercises for the course

"Mathematical modeling in biology"

Sheet 11

Exercise 44: Infinitesimal mean of the general Galton-Watson process Let $(G_t)_{t\geq 0}$ be a Galton-Watson process in continuous time in which each individual branches at rate $\beta \in (0,\infty)$ and leaves $k \in \mathbb{N}_0$ children with probability $p_k \in [0,1]$ where $\sum_{k=0}^{\infty} p_k = 1$. All involved Poisson processes are independent. Determine the infinitesimal mean and the infinitesimal variance of this process.

Exercise 45: Diffusion approximation of a binary Galton-Watson process Recall the binary Galton-Watson process from Exercise 40. Let the initial number $G_0^{(n)} \in \mathbb{N}_0$ of individuals scale with $n \in \mathbb{N}$, that is, $G_0^{(n)}/n \to x \in [0,\infty)$ as $n \to \infty$. What is the diffusion approximation of the rescaled process $\left(G_t^{(n)}/n\right)_{t\geq 0}$ for large $n \in \mathbb{N}_0$?
Hint: Determine the limits of the infinitesimal mean and of the infinitesimal variance of the process $\left(G_t^{(n)}/n\right)_{t\geq 0}$ as $n \to \infty$.

Exercise 46: Diffusion approximation of Kingman's coalescent Let us assume that we have a large sample of order $k \in \mathbb{N}$ and that we want to rescale the coalescent with k. Assume that $C_0^{(k)}/k \to y \in [0,\infty)$ as $k \to \infty$. Moreover assume that the coalescence rate γ_k per pair of individuals scales such that $\gamma_k \cdot k \to c \in (0,\infty)$ as $k \to \infty$. What is the diffusion approximation of the rescaled process $\left(C_t^{(k)}/k\right)_{t\geq 0}$ for large $k \in \mathbb{N}$?

Hint: Recall the infinitesimal mean and the infinitesimal variance of Kingman's coalescent from the lecture. Determine the limits of the infinitesimal mean and of the infinitesimal variance of the process $\left(C_t^{(k)}/k\right)_{t\geq 0}$ as $k \to \infty$.
Remark: One is usually interested in small sample sizes; this exercise is for training purposes only.

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