Exercises for the course

"Mathematical modeling in biology"

Sheet 06

Exercise 24: Blower's model for HIV-progression II Recall Blower's model for HIV-progression from Exercise 16. Now include the possibility of resistance of some virus cells.

Exercise 25: A simple Markov chain A simple minded random walker moves in discrete time between three locations A, B and C with the following probabilities

- From A to B or C with equal probability.
- From B to A twice as often as from B to A.
- Never from C to A. In a quarter of the cases, he stays at C.
- He is never two times in a row in A or in B.

What you should do:

- Draw a flow diagram.
- Then write down the transition matrix.
- Finally check that $\rho = (698)/23$ is an invariant distribution on the tree locations.

Exercise 26: A random walk with drift In the lecture, we had a symmetric random walk on $\mathbb{Z}$ and caculated the approximate position at time n for large $n \in \mathbb{N}$. Now we consider a random walk with drift. Fix $p \in [0,1]$. Let $(R_n)_{n\in\mathbb{N}_0}$ be a Markov process in discrete time as follows:

- $R_0 = 0$
- The random walker takes a step to the right with probability $\frac{1+p}{2}$.

Determine the approximate position of the random walker at large times, that is, determine the approximate distribution of R_n when $n \in \mathbb{N}$ is large. Hint: Apply the central limit theorem.

Exercise 27: Equilibria and stability The following model arises in the context of host-parasite systems where a defense allele is also costly and negative selected. Let $a \in (1,\infty)$, $\theta \in (\frac{1}{a}, \frac{1}{a-1})$ and $s \in (0,\infty)$. Determine the equilibria of the following differential equation

$$\frac{d}{dt}x_t = (a - x_t)\Big(\theta(a - x_t) - 1\Big) - s x_t(1 - x_t)$$

Convince yourself that $x_t \in [0,1]$ for all times $t \in [0,\infty)$. Moreover determine stability of all equilibria. Hint: Do not get lost in calculations. Argue with the type of the function on the right-hand side to see, which points are in $[0,1]$.