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STATISTICS FOR EES AND MEME — EXERCISE SHEET 5

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1. Two dice are tossed. Given that the sum of the pips is 8, what is the probability that at least one of the dice shows 3 pips?

2. Imagine a test for an infectious disease with a sensitivity of 99.9%, i.e. it detects the disease for 99.9% of the patients that have the disease, and a specificity of 90%, i.e. it indicates the disease for 10% of the uninfected patients. If 2% of the population are infected and the tests indicates the disease for a person that was randomly selected from the population, what is then the probability that the person is indeed infected?

3. You have two dice that look exactly the same, but one is fair and the other one is loaded and gives a six in 1/3 of the cases (and other results with a probability of 2/15 each). You forgot which of the two dice is which, so you roll each dice five times. The first dice shows a six only once and with the second dice you obtain two sixes. Given this result, calculate the probability that the first dice is fair.

4. A casino offers the following game: One dice is rolled, and if X is the number of pips, then the player wins $f(X) = 3 - (x-3)^2$ Euros, where negative values of $f(X)$ mean that the player loses money. Let $G = f(X)$ be the gain of a player in one game. Let T be the total gain of a player who played the game 100 times (where a negative gain is actually a loss).

(a) Calculate $\mathbb{E}G$ in two ways:

  (a) once directly with the definition of the expectation value $\sum_y y \cdot \Pr(G = y)$

  (b) once as $\sum_x f(x) \cdot \Pr(X = x)$

(b) Calculate the expectation value and the standard deviation of T.

(c) Which well-known distribution is can be used to approximate the distribution of T?

5.

(a) Imagine a board game in which you always roll a dice when it is your turn and move forward the number of steps corresponding to the number of dots shown on the dice. Calculate the expectation value and the standard deviation of the total number of steps you have moved forward after it has been your turn ten times (assuming that the board is large enough).

(b) Calculate this expectation value and standard deviation also for a variant of the game in which you can roll the dice once more when you had a six and move on accordingly. (But you cannot roll the dice a third time in one turn when you had two sixes.)