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STATISTICS FOR EES AND MEME—EXERCISE SHEET 4

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1. Imagine a test for an infectious disease with a sensitivity of 99.9%, i.e. it detects the disease for 99.9% of the patients that have the disease, and a specificity of 90%, i.e. it indicates the disease for 10% of the uninfected patients. If 2% of the population are infected and the tests indicates the disease for a person that was randomly selected from the population, what is then the probability that the person is indeed infected?

2. Let X and Y be random variables with values in $\{1, 2, 3\}$ and $\{0, 1\}$ and

$$
\begin{aligned}
\Pr(X = 1, Y = 0) &= \frac{1}{3} \qquad \Pr(X = 1, Y = 1) = 0 \\
\Pr(X = 2, Y = 0) &= \frac{1}{4} = \Pr(X = 2, Y = 1) \\
\Pr(X = 3, Y = 0) &= \frac{1}{12} = \Pr(X = 3, Y = 1).
\end{aligned}
$$

Compute

(a) $\Pr(Y = 0)$ and $\Pr(Y = 1)$

(b) $\mathbb{E}X$ and $\mathbb{E}Y$

(c) $\mathbb{E}(X^2)$ and $\mathbb{E}(Y^2)$

(d) $\mathrm{Var}(X)$ and $\mathrm{Var}(Y)$

(e) $\mathrm{Cov}(X, Y)$

(f) $\mathrm{Cor}(X, Y)$

3. Proof the following formulas or find counter examples:

(a) $\mathbb{E}(f(X)) = \sum_{x \in \mathcal{S}} f(x) \cdot \Pr(X = x)$

(b) Var$(X) = \mathbb{E}(X^2) - (\mathbb{E}X)^2$

(c) Var$(X \cdot Y) =$ Var$(X)\cdot$Var(Y)

(d) Cov$(a \cdot X, Y + Z) = a\cdot$Cov$(X, Y) + a\cdot$Cov$(X, Z)$

4. Let $X_1, X_2, \ldots, X_n$ be independent random variables with the same distribution with finite variance σ^2. Proof that

$$
\mathbb{E}\left(\frac{1}{n-1}\sum_{i=1}^{n}\left(X_i - \frac{1}{n}\sum_{j=1}^{n} X_j\right)^2\right) = \sigma^2.
$$